Developing Computational Fluency

Please read the front side of the pink handout at your table before we begin – it's an introduction to the day.

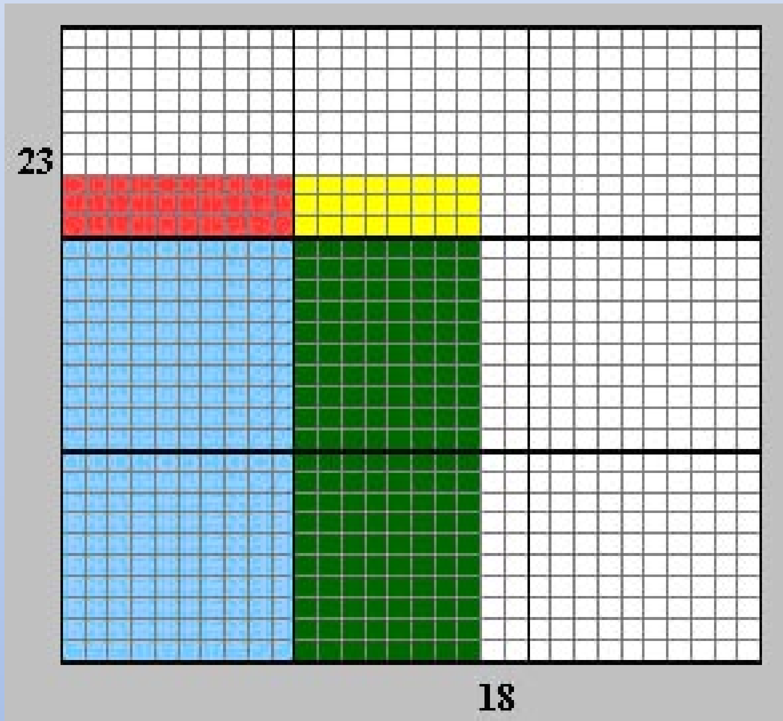
What is the problem?

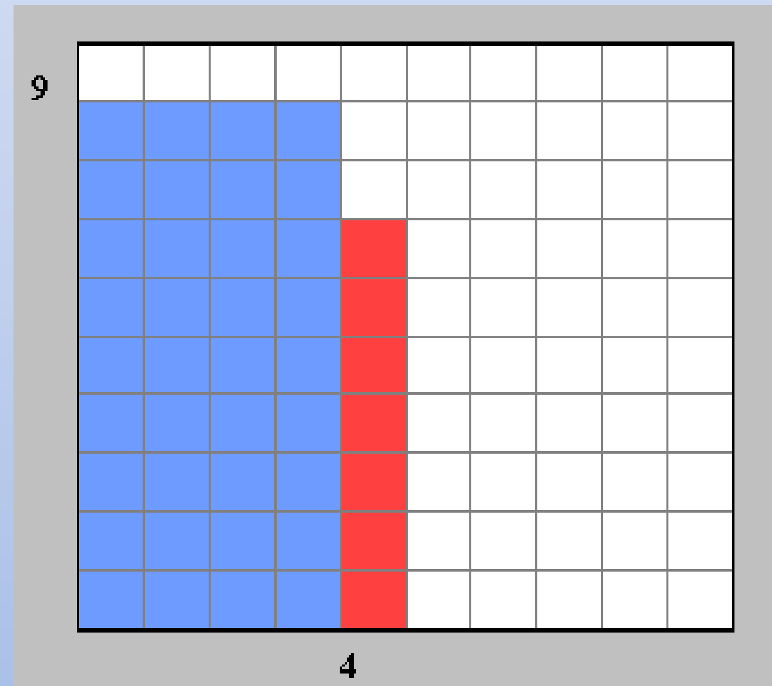
$$\begin{array}{r} 112 \\ +\ 40 \\ \hline 512 \end{array}$$

$$\begin{array}{r} {\scriptstyle 2} \\ 57 \\ \times\ 4 \\ \hline 288 \end{array}$$

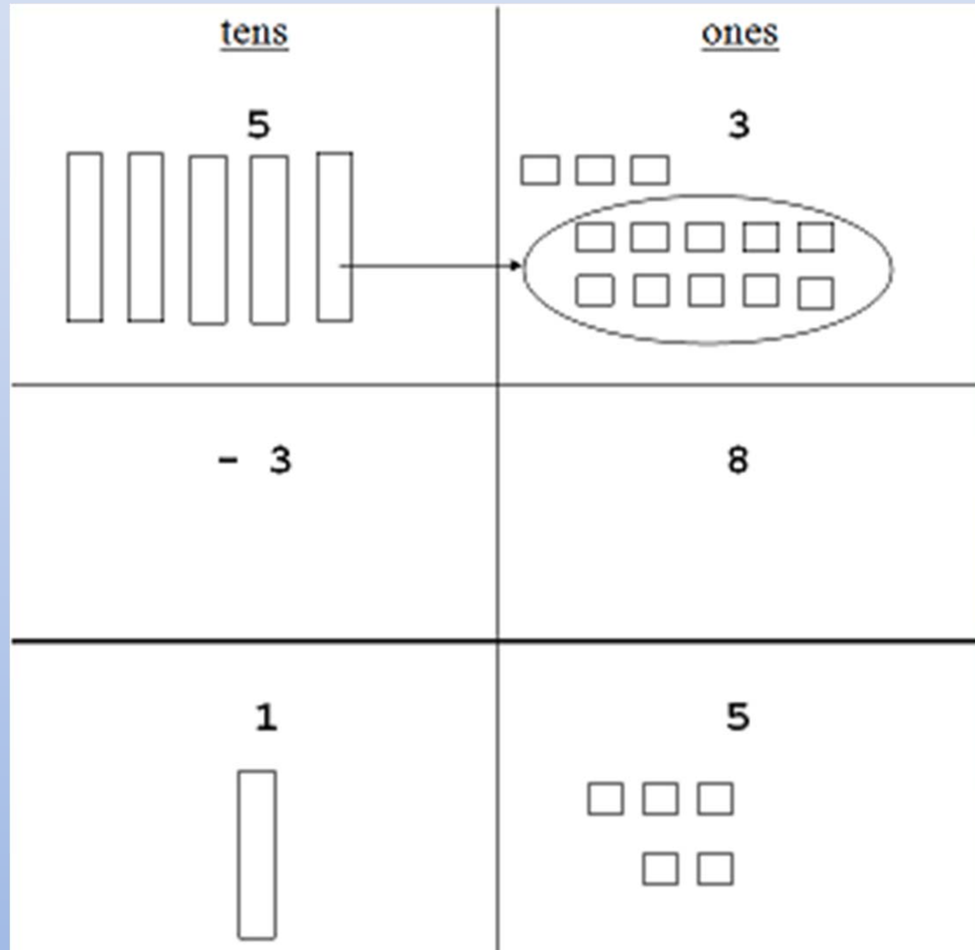
From **Developing Computational Fluency with Whole Numbers in the Elementary Grades**, Susan Jo Russell

What do these represent?





How important is this?



$$\begin{array}{r} {\scriptstyle 4\ 13} \\ \cancel{5}3 \\ -38 \\ \hline 15 \end{array}$$

Efficiency, Accuracy, and Flexibility

Efficiency

- *Efficiency* implies that the student does not get bogged down in too many steps or lose track of the logic of the strategy. An efficient strategy is one that the student can carry out easily, keeping track of subproblems and making use of intermediate results to solve the problem.

Developing Computational Fluency with Whole Numbers in the Elementary Grades, by Susan Jo Russell

Accuracy

- *Accuracy* depends on several aspects of the problem-solving process, among them careful recording, knowledge of number facts and other important number relationships, and double-checking results.

Flexibility

- *Flexibility* requires the knowledge of more than one approach to solving a particular kind of problem, such as two-digit multiplication. Students need to be flexible in order to choose an appropriate strategy for the problem at hand, and also to use one method to solve a problem and another method to double-check the results.

More than memorization of a single procedure

1. An understanding of the meaning of the operations and their relationships to each other – for example, the inverse relationship between multiplication and division;

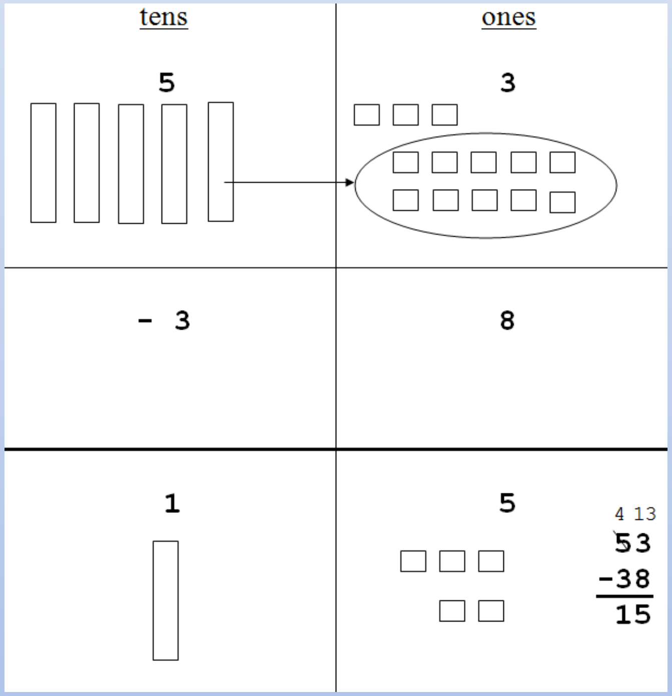
$54 \div 9 = \_\_\_$ $9 \times \_\_\_ = 54$

2. the knowledge of a large repertoire of number relationships, including the addition and multiplication “facts” as well as other relationships, such as how 4 x 5 is related to 4 x 50; and

$$4 \times 50 = 4 \times (5 \times 10) = 20 \times 10 = 200$$

$$\begin{array}{r} 1002 \\ -\ 998 \\ \hline \end{array}$$

3. a thorough understanding of the base ten number system, how numbers are structured in this system, and how the place value system of numbers behaves in different operations – for example, that 24 + 10 = 34 or 24 x 10 = 240.



Addition and Multiplication "Facts"

Basic Approach (part 1):

- Children learn the meaning of addition and subtraction in K-1 through the use of well-structured problems (CGI).
- Multiplication and division concepts are developed in this way in 3^{rd} grade.

Problem sets, video

- Most number combinations are learned through repeated problem solving when children are motivated to use more efficient strategies over time.

  7 birds were sitting in a tree. 6 more birds flew up to the tree. How many birds were there altogether in the tree?

Basic Approach (part 2):

- Instruction on strategies is helpful for many students
  - 1) They develop their own
  - 2) They are taught

Developing their own

- Use simple story problems designed in such a manner that students are most likely to develop a strategy as they solve it.
- Manipulatives and drawing materials should be available. On-going work with five- and ten-frame cards is helpful.

CGI examples of story problems using 6+7 and 8+5

Teaching specific strategies

- A lesson may revolve around a special collection of facts for which a particular type of strategy is appropriate.
- The class can discuss how these facts might all be alike in some way, producing the strategy, or
- The teacher might suggest an approach and see if students are able to use it on similar facts.

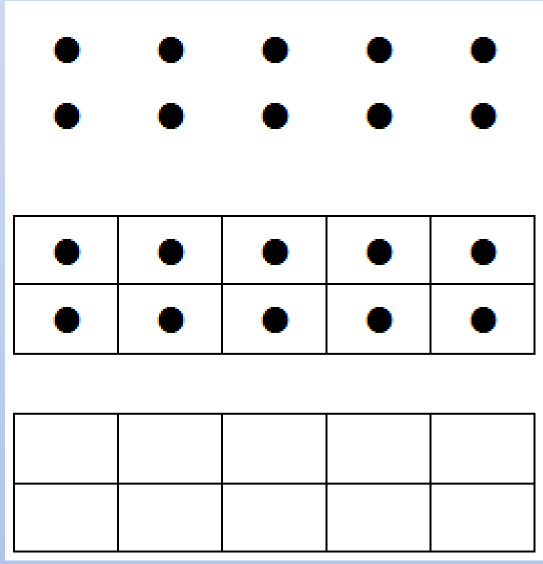
From Van de Walle and Lovin, 2006.
See the Math Facts 2nd grade video.

- There is a huge temptation simply to tell students about a strategy and then have them practice it. Though this can be effective for some students, many others will not personally relate to your ideas or may not be ready for them.
- Continue to discuss strategies invented in your class and plan lessons the encourage strategies.

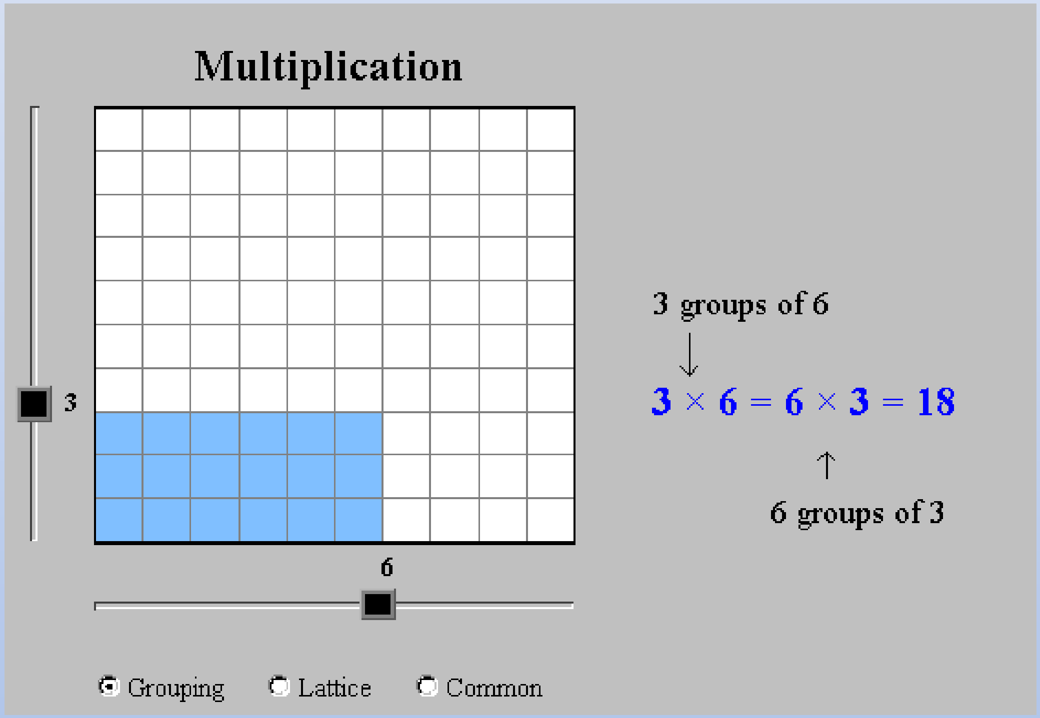
U-F-G Framework

Conceptual Understanding	Fluency	Generalization
Addition is putting together and adding to, subtraction is taking apart, taking from and comparing.		
Problems are solved using objects and drawings to represent situations.	Fluency starts with strategies such as counting on, making ten, doubles plus one.	
	By end of 2nd grade, know all sums from memory.	Addition is the foundation for multiplication through skip counting of arrays.

Arrays to Area Models



Area Models



- Make a visual model of 36 ÷ 9
- Make up two word problems, one where 9 stands for the number of objects in each group, and one where 9 stands for the number of groups.

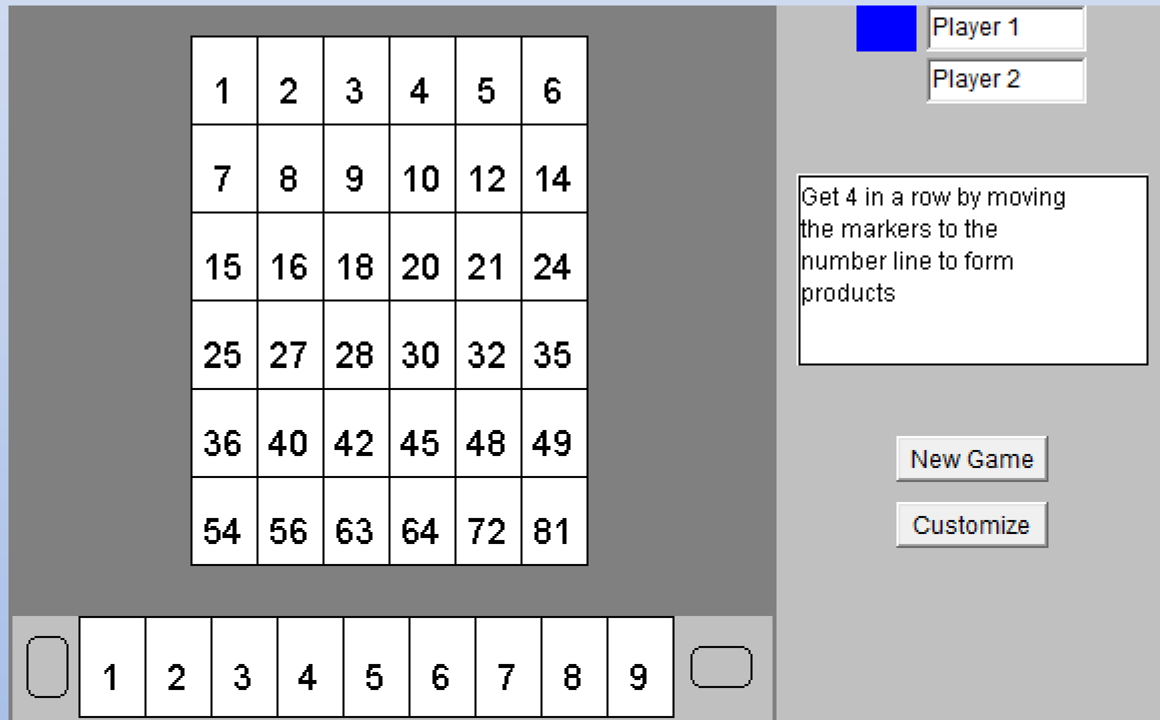
- Number talks 3rd grade 7 x 7

Basic Approach (part 3): For students in 4th or 5th grade who don't have command of all the combinations

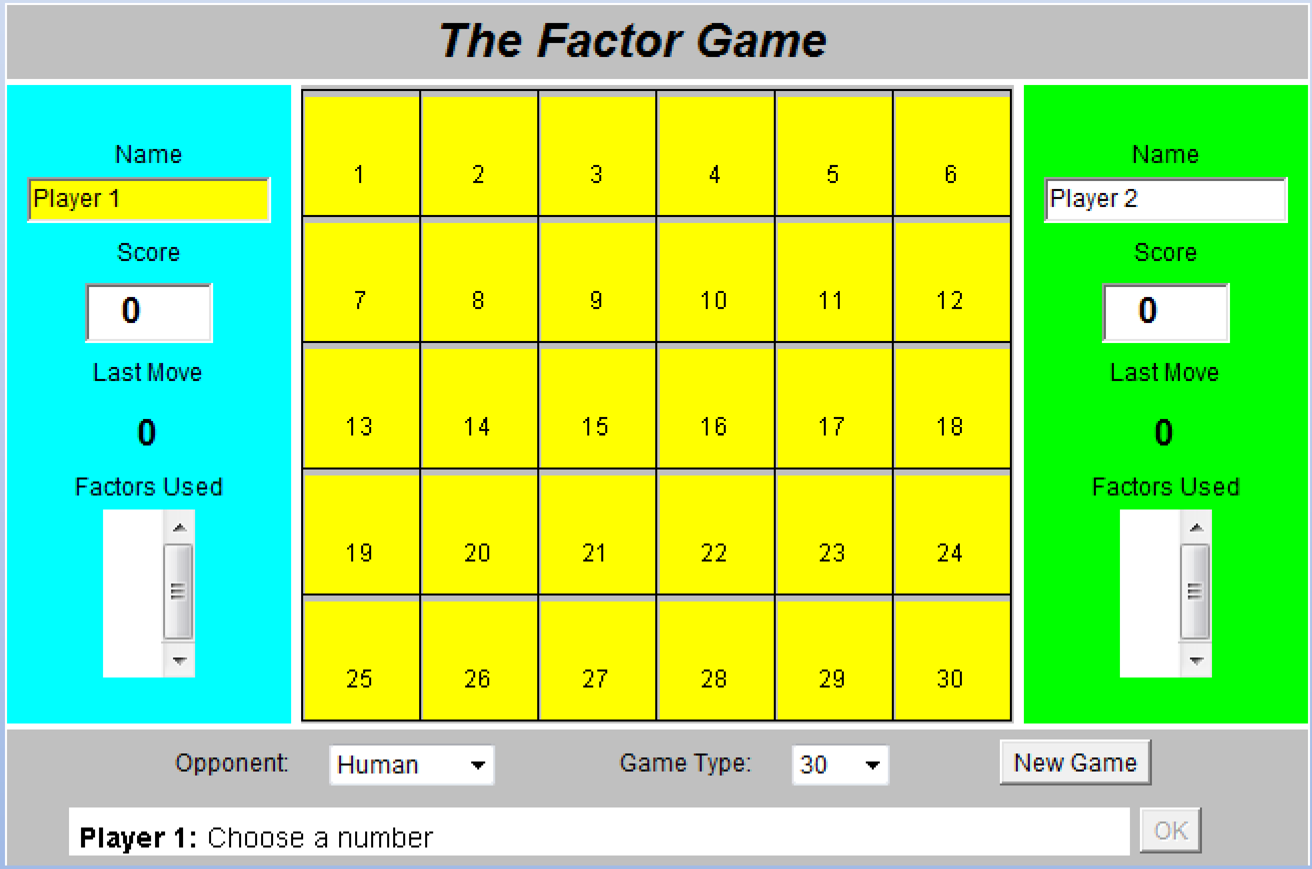
- Competitive games and non-boring practice are helpful for developing strategies, leading to quick retrieval.
- Both immediate, focused practice and cumulative practice.

On-line games, card games, board games

The Product Game



The Factor Game



What intervention programs are available to support this?

Multi-digit Operations

- Can students reason about the problems first?

$$\begin{array}{r} 112 \\ +\ 40 \\ \hline 512 \end{array}$$

$$\begin{array}{r} {}^{2} \\ 57 \\ \times\ 4 \\ \hline 288 \end{array}$$

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Number Talks 2nd grade 38 + 37

Base Ten Concepts

Using objects grouped by ten:

- There are 10 popsicle sticks in each of these 5 bundles, and 3 loose popsicle sticks. How many popsicle sticks are there all together?
  - Students' strategies?
- The extension: The teacher puts out one more bundle of ten popsicle sticks and asks students "Now how many popsicle sticks are there all together?" What strategies would students use to answer this?

Multi-digit Progression

1. Mental strategies
2. Place value representations (e.g. base 10 blocks, pictures)
3. Algorithms

Multi-digit Problems

1. Separating, result unknown

Peter had 28 cookies. He ate 13 of them. How many did he have left? Write this as a number sentence: 28 – 13 = ____

There were 51 geese in the farmer's field. 28 of the geese flew away. How many geese were left in the field?

2. Comparing two amounts (height, weight, quantity)

There are 18 girls on a soccer team and 5 boys. How many more girls are there than boys on the soccer team?

Multi-digit Problems

3. Part-whole where a part is unknown

There are 23 players on a soccer team. 18 are girls and the rest are boys. How many boys are on the soccer team?

4. Distance between two points on a number line (difference in age, distance between mileposts)

How far is it on the number line between 27 and 42?

Multi-digit Problems

- There were 51 geese in the farmer's field. 28 of the geese flew away. How many geese were left in the field?
- There were 28 girls and 35 boys on the playground at recess. How many children were there on the playground at recess?
- Misha has 34 dollars. How many dollars does she have to earn to have 47 dollars?
  - What strategies can you come up with?
  - Counting single units. Direct modeling with tens and ones. Invented algorithms: Incrementing by tens and then ones, Combining tens and ones, Compensating.

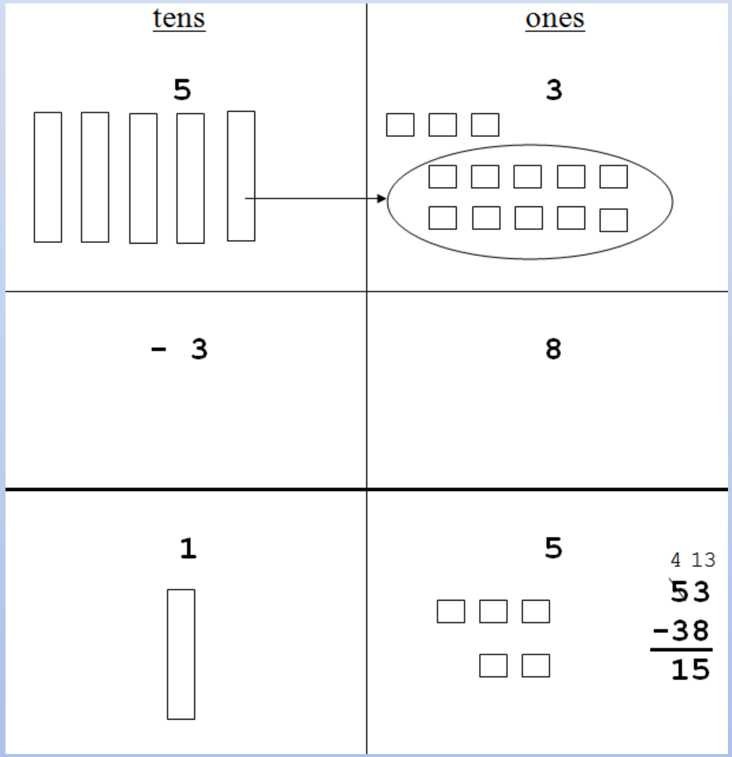
Development of Algorithms

- The C-R-A approach is used to develop meaning for algorithms.
- Without meaning, students can't generalize the algorithm to more complex problems.

C: Concrete materials (used for counting by 10's and 1's)

R: Visual representations

A: Abstract algorithm



Often students in need of extra support require explicit instruction to make these connections.

Typical Learning Problems

1. $\begin{array}{r} 34 \\ -\ 2 \\ \hline 32 \end{array}$	2. $\begin{array}{r} 86 \\ -\ 7 \\ \hline 81 \end{array}$	3. $\begin{array}{r} 71 \\ -69 \\ \hline 18 \end{array}$
4. $\begin{array}{r} 42 \\ -27 \\ \hline 25 \end{array}$	5. $\begin{array}{r} 56 \\ -51 \\ \hline 5 \end{array}$	6. $\begin{array}{r} 854 \\ -\ 60 \\ \hline 814 \end{array}$
7. $\begin{array}{r} 305 \\ -147 \\ \hline 242 \end{array}$	8. $\begin{array}{r} 832 \\ -807 \\ \hline 35 \end{array}$	9. $\begin{array}{r} 420 \\ -119 \\ \hline 319 \end{array}$

1. $\begin{array}{r} {}^{2}\ \ {}^{1} \\ \cancel{3}\,4 \\ -\ \ 2 \\ \hline 212 \end{array}$

2. $\begin{array}{r} {}^{7}\ \ {}^{1} \\ \cancel{8}\,6 \\ -\ \ 7 \\ \hline 79 \end{array}$

3. $\begin{array}{r} {}^{6}\ \ {}^{1} \\ \cancel{7}\,1 \\ -69 \\ \hline 2 \end{array}$

4. $\begin{array}{r} {}^{3}\ \ {}^{1} \\ \cancel{4}\,2 \\ -27 \\ \hline 15 \end{array}$

5. $\begin{array}{r} {}^{4}\ \ {}^{1} \\ \cancel{5}\,6 \\ -51 \\ \hline 115 \end{array}$

6. $\begin{array}{r} {}^{7}\ {}^{14}\ {}^{1} \\ \cancel{8}\,\cancel{5}\,4 \\ -\ \ 60 \\ \hline 7814 \end{array}$

7. $\begin{array}{r} {}^{2}\ {}^{9}\ {}^{1} \\ \cancel{3}\,\cancel{0}\,5 \\ -147 \\ \hline 158 \end{array}$

8. $\begin{array}{r} {}^{7}\ {}^{12}\ {}^{1} \\ \cancel{8}\,\cancel{3}\,2 \\ -807 \\ \hline 1125 \end{array}$

9. $\begin{array}{r} {}^{3}\ {}^{11}\ {}^{1} \\ \cancel{4}\,\cancel{2}\,0 \\ -119 \\ \hline 2101 \end{array}$

1. $\begin{array}{r} 34 \\ -\ \ 2 \\ \hline 32 \end{array}$

2. $\begin{array}{r} {}^{7}\cancel{8}\,{}^{1}6 \\ -\ \ 7 \\ \hline 79 \end{array}$

3. $\begin{array}{r} {}^{6}\cancel{7}\,{}^{1}1 \\ -69 \\ \hline 2 \end{array}$

4. $\begin{array}{r} {}^{3}\cancel{4}\,{}^{1}2 \\ -27 \\ \hline 15 \end{array}$

5. $\begin{array}{r} 56 \\ -51 \\ \hline 5 \end{array}$

6. $\begin{array}{r} {}^{7}\cancel{8}\,{}^{1}5\,4 \\ -\ \ 6\,0 \\ \hline 794 \end{array}$

7. $\begin{array}{r} {}^{1}\cancel{3}\,{}^{1}0\,{}^{1}5 \\ -147 \\ \hline 68 \end{array}$

8. $\begin{array}{r} {}^{7}\cancel{8}\,3\,{}^{1}2 \\ -807 \\ \hline \cancel{35} \\ 25 \end{array}$

9. $\begin{array}{r} {}^{3}\cancel{4}\,2\,{}^{1}0 \\ -119 \\ \hline 211 \end{array}$

Multidigit Multiplication

- What are the “pieces” of multidigit multiplication as represented by problems on the diagnostic assessment?

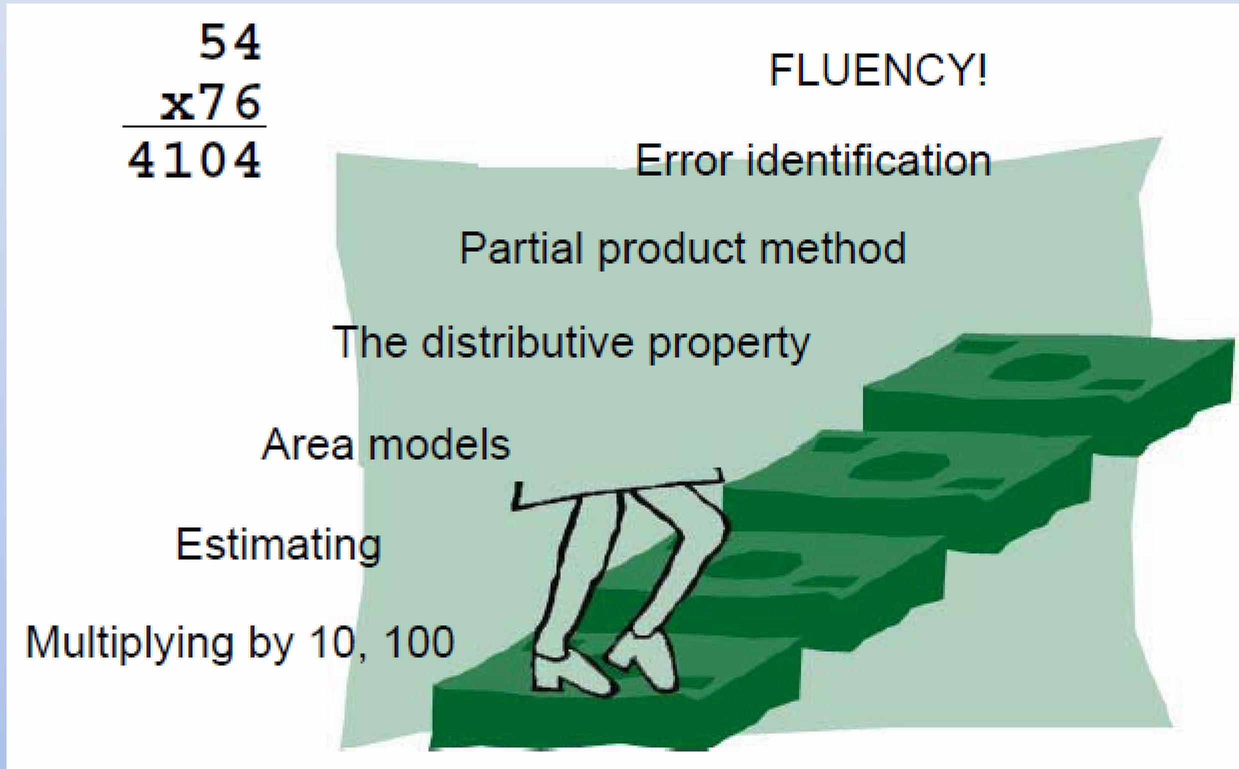
What students need to know and be able to do:

- know that the concept of multiplication is repeated adding or skip counting – finding the total number of objects in a set of equal size groups
- be able to represent situations involving groups of equal size with objects, words and symbols.
- know multiplication combinations fluently (which may mean some flexible use of derived strategies).
- know how to multiply by 10 and 100.

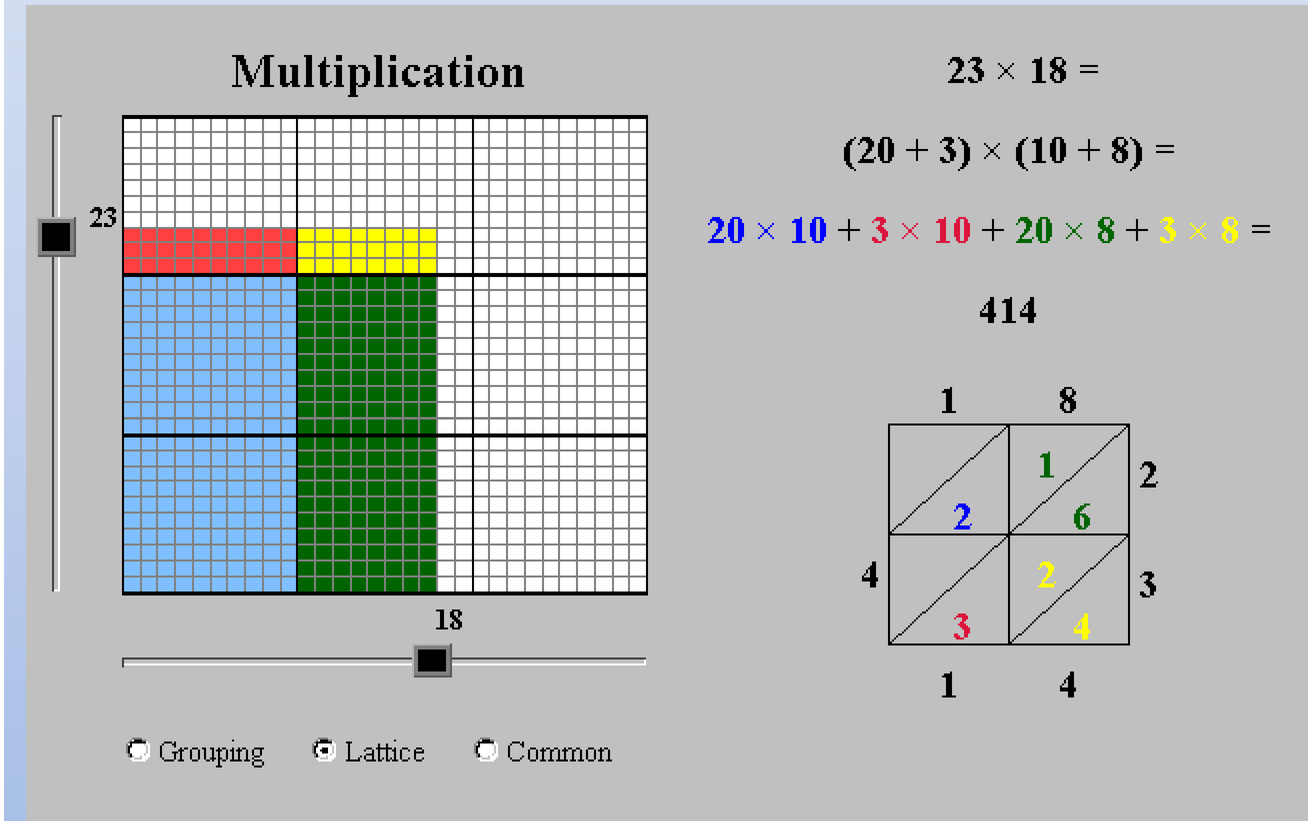
Continued…

What students need to know and be able to do:

- use number sense to estimate the result of multiplying.
- use area and array models to represent multiplication and to simplify calculations.
- understand how the distributive property works and use it to simplify calculations
- use alternative algorithms like the partial product method (based on the distributive property) and the lattice method.


54
x76
4104
FLUENCY!
Error identification
Partial product method
The distributive property
Area models
Estimating
Multiplying by 10, 100

Visual and symbolic representations of multiplication using distributive prop.



Distributive property:

$23 \times 18 = 20(10 + 8) + 3(10 + 8)$

The partial product method:

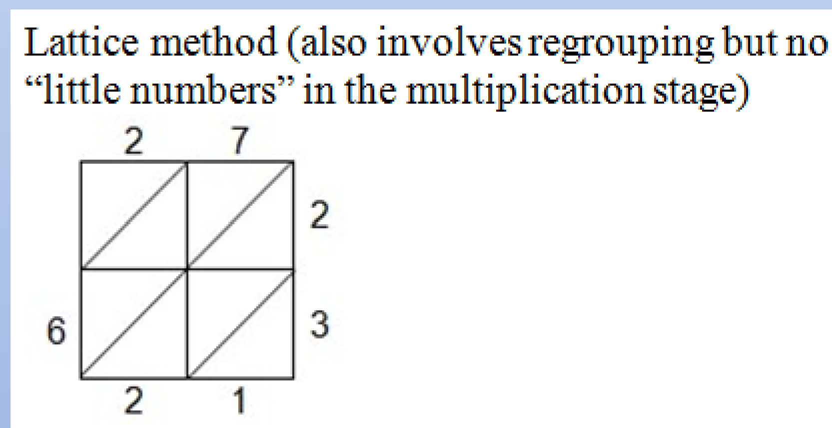
18 x 23	10	8	
20	200	160	360
3	30	24	54
			414

$$\begin{array}{r} 23 \\ \times\ 18 \\ \hline 24 \\ 160 \\ 30 \\ 200 \\ \hline 414 \end{array}$$

Rectangle Multiplication at the National Library of Virtual Manipulatives

Make up your own example.

- Chinese teachers often focus on just 2x2 multiplication to ensure that students understand what’s going on with place value.



What does the empty spot mean?

$$
\begin{array}{rrr}
 & {\scriptstyle 2} & \\
 & 2 & 3 \\
\times & 1 & 8 \\
\hline
1 & 8 & 4 \\
2 & 3 & \\
\hline
4 & 1 & 4
\end{array}
$$

$$
\begin{array}{rrr}
 & 2 & 3 \\
\times & 1 & 8 \\
\hline
 & 2 & 4 \\
1 & 6 & 0 \\
 & 3 & 0 \\
2 & 0 & 0 \\
\hline
4 & 1 & 4
\end{array}
$$

Try 45 x 23 –
first estimate

“This kind of teaching leads students not to *memorizing,* but to the development of *mathematical memory* (Russell, 1999). Important mathematical procedures cannot be ‘forgotten over the summer,’ because they are based in a web of connected ideas about fundamental mathematical relationships.”

Online Resources

1. http://inghamisd.org
2. Find Out More About: Wiki Spaces
3. Elementary Math Resources
4. 4th-5th Grade